THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH2230 Complex Variables with Applications
2024-2025 T2

Homework 8
Due time: 19th March 2025, 23:59

All problems are taken from COMPLEX VARIABLES AND APPLICATIONS, Ninth Edition, by James Ward Brown/Ruel V. Churchill.

P.196: 2, 4, 8, 11
P.218: 1

Practice Problems (Do not turn in)

P.219: 4, 6, 8

Please also find the Homework problems below and email me (xiaolilin@cuhk.edu.hk) if there is any typo.

1. (**P.196, Q2**) Obtain the Taylor series

$$e^z = e\sum_{n=0}^{\infty} \frac{(z-1)^n}{n!} \quad (|z-1| < \infty)$$

for the function $f(z) = e^z$ by

(a) using $f^{(n)}(1)\,(n = 0,\, 1,\, 2 \cdots)$;
(b) writing $e^z = e^{z-1}e$.

2. (**P.196, Q4**)

With the aid of the identity

$$\cos z = -\sin(z - \frac{\pi}{2}),$$

expand $\cos z$ into a Taylor series about the point $z_0 = \pi/2$..

3. (**P.196, Q8**)

   Rederive the Maclaurin series (4) in Sec. 64 for the function $f(z) = \cos z$ by

   (a) using the definition

   $$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

   in Sec.37 and appealing to the Maclaurin series (2) for e^z in Sec. 64;

   (b) showing that

   $$f^{(2n)}(0) = (-1)^n, \quad \text{and} \quad f^{(2n+1)}(0) = 0 \quad (n = 0,\, 1,\, 2\, \cdots)$$

4. (**P.197, Q11**) Show that when $0 < |z| < 4$,

   $$\frac{1}{4z - z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{n+2}}.$$

5. (**P.218, Q1**) By differentiating the Maclaurin series representation

   $$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \quad (|z| < 1),$$

   obtain the expansions

   $$\frac{1}{(1-z)^2} = \sum_{n=0}^{\infty} (n+1) z^n \quad (|z| < 1)$$

   and

   $$\frac{2}{(1-z)^3} = \sum_{n=0}^{\infty} (n+1)(n+2) z^n \quad (|z| < 1).$$